Numerical Methods

Unit - 2 Interpolation and Approximation

2 Mark Questions

1. State Lagranges interpolation formula

Solution :-

Let $y = f(x)$ be a function which takes the values $y_0, y_1, \dots y_n$ and corresponding to $x_0, x_1, \dots x_n$

The Lagranges interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} \cdot y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} \cdot y_1 + \dots\dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} \cdot y_n$$

2. What is the Lagranges formula to find y, if three sets of values $(x_0, y_0), (x_1, y_1)$ and (x_2, y_2) are given

Solution :-

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \cdot y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \cdot y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \cdot y_2$$

3. What is the assumption we make when Lagranges formula is used?

Solution :-

Lagranges interpolation formula can be used whether the values of x, the independent variable are equally spaced or not whether the difference of y become smaller or not.

4. Using Newton's divided difference formula find the missing value from the table.

x : 1 2 4 5 6

y : 14 15 5 - 9

Solution :

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	14	$\frac{15-14}{2-1} = 1$		
2	15	$\frac{5-15}{4-2} = -5$	$\frac{-5-1}{4-1} = -2$	
4	5	$\frac{9-5}{6-4} = 2$	$\frac{2+5}{6-2} = \frac{7}{4}$	$\frac{7/4+2}{6-1} = \frac{3}{4}$
6	9			

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + \dots$$

$$= 14 + (x-1)(1) + (x-1)(x-2)(-2) + (x-1)(x-2)(x-4)\left(\frac{3}{4}\right)$$

$$= 14 + x - 1 - 2(x-1)(x-2) + \frac{3}{4}(x-1)(x-2)(x-4)$$

$$f(5) = 13 + 5 - 2(4)(3) + \frac{3}{4}(4)(3)(1)$$

$$= 18 - 24 + 9$$

$$= 3$$

5. Using Newton's divided difference formula determine $f(3)$ from the data

x : 0 1 2 4 5

f(x) : 1 14 15 5 6

Solution :

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	1	$\frac{14-1}{1-0} = 13$			
1	14		$\frac{1-13}{2-0} = -6$	$\frac{-2+6}{4-0} = 1$	
2	15	$\frac{15-14}{2-1} = 1$	$\frac{-5-1}{4-1} = -2$		$\frac{1-1}{5-0} = 0$
4	5	$\frac{5-15}{4-2} = -5$		$\frac{2+2}{5-1} = 1$	
5	6	$\frac{6-5}{5-4} = 1$	$\frac{1+5}{5-2} = 2$		

By Newtons divided difference interpolation formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + \dots$$

$$= 1 + (x-0)13 + (x-0)(x-1)(-6) + (x-0)(x-1)(x-2)(1) + 0$$

$$= 1 + 13x - 6x(x-1) + x(x-1)(x-2)$$

$$f(3) = 1 + 39 - 6(3)(2) + 3(3-1)(3-2)$$

$$= 40 - 36 + 3(2)(1)$$

$$= 40 - 36 + 6$$

$$= 10.$$

6. Derive Newtons backward difference formula by using operator method

Solution :

$$P_n(x) = P_n(x_n + vh)$$

$$= E^v P_n(x_n)$$

$$= (1-\nabla)^{-v} y_n \quad \text{since } E = (1-\nabla)^{-1}$$

$$= \left[1 + v\nabla + \frac{v(v+1)}{2!}\nabla^2 + \frac{v(v+1)(v+2)}{3!}\nabla^3 + \dots\right] y_n$$

$$= y_n + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \dots$$

where $v = \frac{x - x_n}{h}$.

7. When Newton's backward interpolation formula is used

Solution:

The formula is used mainly to interpolate the values of y near the end of a set of tabular values and also for extrapolating the values of y a short distance ahead (to the right) of y_0

8. Newtons forward interpolation formula used only for __ intervals.

Solution:

Equidistant (or) equal intervals.

9. Write the newtons forward and backward formula:

Solution:

Newtons forward formula

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

where $p = \frac{x - x_0}{h}$

Newtons Backward formula

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

Where $p = \frac{x - x_n}{h}$.

10. Write the Inverse Lagrangian Formula:-

Solution:

$$x = \frac{(y-y_1)(y-y_2)\cdots(y-y_n)}{(y_0-y_1)(y_0-y_2)\cdots(y_0-y_n)} \cdot x_0 + \frac{(y-y_0)(y-y_2)\cdots(y-y_n)}{(y_1-y_0)(y_1-y_2)\cdots(y_1-y_n)} \cdot x_1 + \cdots$$

11. Write the cubic spline interpolation formula

Solution:

$$S_{i-1} + 4S_i + S_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right].$$

For $i = 1, 2, 3 \ldots n-1$.

For natural ~~nature~~ cubic spline,

$$S(x) = \frac{1}{6h}\left[(x_i - x)^3 S_{i-1} + (x - x_{i-1})^3 S_i\right] + \frac{1}{h}\left[(x_i - x)\left(y_{i-1} - \frac{h^2}{6}S_{i-1}\right)\right] + \frac{1}{h}\left[(x - x_{i-1})\left(y_i - \frac{h^2}{6}S_i\right)\right].$$